Towards Object Orientation in Agda
Part I: Coalgebras and IO

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Coalgebras in Agda

Interactive Programs in Agda

State-Dependent IO

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Codata Type

- Idea of Codata Types:

$$\begin{array}{l} \textsf{codata Stream} : \textsf{Set where} \\ \quad \textsf{cons} \quad : \quad \mathbb{N} \to \textsf{Stream} \to \textsf{Stream} \end{array}$$

- Same definition as inductive data type but we are allowed to have infinite chains of constructors

$$\textsf{cons}\ n_0\ (\textsf{cons}\ n_1\ (\textsf{cons}\ n_2\ \cdots))$$

- **Problem 1:** Non-normalisation.
- **Problem 2:** Equality between streams is equality between all elements, and therefore undecidable.
- **Problem 3:** Underlying assumption is

$$\forall s : \textsf{Stream}. \exists n, s'. s = \textsf{cons}\ n\ s'$$

which results in undecidable equality.

Solution: Coalgebras Defined by Observations

- We define coalgebras by their observations. Tentative syntax

$$\begin{array}{l} \text{coalg Stream} : \text{Set where} \\ \quad \begin{array}{lcl} \text{head} & : & \text{Stream} \to \mathbb{N} \\ \text{tail} & : & \text{Stream} \to \text{Stream} \end{array} \end{array}$$

- Stream is the largest set of terms which allow arbitrary many applications of tail followed by head to obtain a natural numbers.
- From this one can develop a general model for coalgebras (see our paper [Set16]).
- Therefore no infinite expansion of streams:
  – for each expansion of a stream one needs one application of tail.

Syntax in Agda

- In Agda the record type has been reused for defining coalgebras:

```
record Stream (A : Set) : Set where
  coinductive
  field
    head : A
    tail : Stream A
```

const and inc can be defined with the syntax as given before

Principle of Guarded Recursion

- Define

$$\begin{array}{llllll} f : A \to \mathrm{Stream} \\ \mathrm{head} & (f\ \ a) & = & \cdots & : & \mathbb{N} \\ \mathrm{tail} & (f\ \ a) & = & \cdots & : & \mathrm{Stream} \end{array}$$

  where

$$\begin{array}{llllll} \mathrm{tail}\,(f\ a) & = & f\ a' & \text{for some} & a' & : & A \\ \text{or} \\ \mathrm{tail}\,(f\ a) & = & s' & \text{for some} & s' & : & \mathrm{Stream}\ \text{given before} \end{array}$$

- **No** function can be applied to the corecursion hypothesis.
- Using sized types one can apply size preserving or size increasing functions to co-IH (Abel).
- Above is example of **copattern matching**.

Example

- Constant stream of $a, a, a, \ldots$

$$\begin{array}{lcl} \text{const} : \{A : \text{Set}\} \to A \to \text{Stream}\ \ A \\ \text{head}\ (\text{const}\ a) & = & a \\ \text{tail}\ \ (\text{const}\ a) & = & \text{const}\ \ a \end{array}$$

- The increasing stream $n, n+1, n+2, \ldots$

$$\begin{array}{lcl} \text{inc} : \mathbb{N} \to \text{Stream}\ \ \mathbb{N} \\ \text{head}\ (\text{inc}\ n) & = & n \\ \text{tail}\ \ (\text{inc}\ n) & = & \text{inc}\ (n+1) \end{array}$$

- Cons is **defined**:

$$\begin{array}{lcl} \text{cons}\ \ : X \to \text{Stream}\ \ X \to \text{Stream}\ \ X \\ \text{head}\ \ (\text{cons}\ \ x\ l) & = & x \\ \text{tail}\ \ (\text{cons}\ \ x\ l) & = & l \end{array}$$

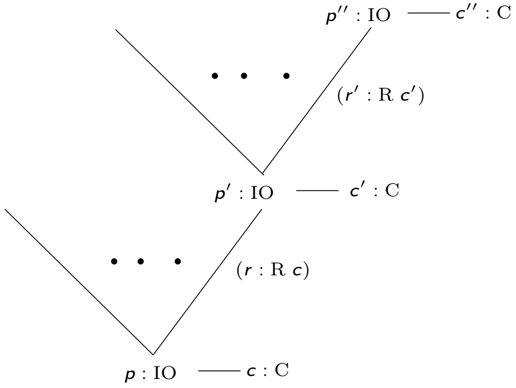
Nested Patter/Copattern Matching

- We can even define functions by a combination of pattern and copattern matching and nest those:
  The following defines the stream

$$\mathsf{stutterDown}\ \ n\ n \ \ = \ \ n, n, n-1, n-1, \ldots 0, 0, n, n, n-1, n-1, \ldots$$

$$
\begin{array}{ll}
\mathsf{stutterDown} : \mathbb{N} \to \mathbb{N} \to \mathsf{Stream}\ \mathbb{N} & \\
\mathsf{head}\ (\mathsf{stutterDown}\ n\ m) & = m \\
\mathsf{head}\ (\mathsf{tail}\ (\mathsf{stutterDown}\ n\ m)) & = m \\
\mathsf{tail}\ (\mathsf{tail}\ (\mathsf{stutterDown}\ n\ (\mathsf{suc}\ m))) & = \mathsf{stutterDown}\ n\ m \\
\mathsf{tail}\ (\mathsf{tail}\ (\mathsf{stutterDown}\ n\ 0)) & = \mathsf{stutterDown}\ n\ n
\end{array}
$$

IO-Trees (Non-State Dependent)



IOInterface

An IOInterface is a record having fields Command and Response:

```
record IOInterface : Set₁ where
  field Command  : Set
        Response : Command → Set
```

Console Interface

```
data ConsoleCommand : Set where
  getLine   : ConsoleCommand
  putStrLn : String → ConsoleCommand

ConsoleResponse : ConsoleCommand → Set
ConsoleResponse getLine        = String
ConsoleResponse (putStrLn s) = Unit

ConsoleInterface : IOInterface
Command ConsoleInterface = ConsoleCommand
Response ConsoleInterface  = ConsoleResponse
```

IO

The set of IO programs IO∞ is the coalgebra having as observation an element of IO.
Elements of IO are IO trees which can have leaves (introduced by return) and nodes (introduced by do):

```
mutual
  record IO∞ (I : IOInterface) (A : Set) : Set where
    coinductive
    field force : IO I A

  data IO (I : IOInterface) (A : Set) : Set where
    do : (c : Command I) (f : Response I c → IO∞ I A)
                           → IO I A
    return : A → IO I A
```

Monadic bind is used to combine programs:

$$
\begin{array}{l}
\mathsf{mutual} \\
\quad \_\gg=\_ : \forall\{A\ B\}\ (m : \mathsf{IO}\ I\ A)\ (k : A \to \mathsf{IO}\infty\ I\ B) \to \mathsf{IO}\ I\ B \\
\quad \mathsf{do}\ c\ f \qquad \gg= k = \mathsf{do}\ c\ \lambda\ x \to f\ x \gg=\infty\ k \\
\quad \mathsf{return}\ a \qquad \gg= k = \mathsf{force}\ (k\ a) \\
\\
\quad \_\gg=\infty\_ : \forall\{A\ B\}\ (m : \mathsf{IO}\infty\ I\ A)\ (k : A \to \mathsf{IO}\infty\ I\ B) \\
\qquad\qquad\qquad \to \mathsf{IO}\infty\ I\ B \\
\quad \mathsf{force}\ (m \gg=\infty\ k) = \mathsf{force}\ m \gg= k
\end{array}
$$

Running Interactive Programs

```
{-# NON_TERMINATING #-}
translateIO : ∀ {A} (tr : (c : C) → NativeIO (R c)) → IO∞ I A
                → NativeIO A
translateIO tr m = case (force m) of λ
  { (do c f)    → (tr c) native>>= λ r → translateIO tr ( f r)
  ; (return a) → nativeReturn a
  }
```

Non termination is unproblematic since this function is only used as part of the compilation process.

Console IO

$$
\begin{array}{l}
\mathrm{IOConsole} : \mathrm{Set} \to \mathrm{Set} \\
\mathrm{IOConsole} = \mathrm{IO}\infty\ \mathrm{ConsoleInterface}
\end{array}
$$

$$
\begin{array}{ll}
\mathrm{translateIOConsoleLocal} : & (c : \mathrm{ConsoleCommand}) \\
 & \to \mathrm{NativeIO}\ (\mathrm{ConsoleResponse}\ c) \\
\mathrm{translateIOConsoleLocal}\ (\mathrm{putStrLn}\ s) & = \mathrm{nativePutStrLn}\ s \\
\mathrm{translateIOConsoleLocal}\ \mathrm{getLine} & = \mathrm{nativeGetLine}
\end{array}
$$

$$
\begin{array}{l}
\mathrm{translateIOConsole} : \{A : \mathrm{Set}\} \to \mathrm{IOConsole}\ A \to \mathrm{NativeIO}\ A \\
\mathrm{translateIOConsole} = \mathrm{translateIO}\ \mathrm{translateIOConsoleLocal}
\end{array}
$$

A First Interactive Program

```
cat : IOConsole Unit
force cat = do getLine λ line →
            do∞ (putStrLn line) λ _ →
            cat
```

- This program doesn't termination check because in guarded recursion we are not allowed to apply the defined function do∞o to the corecursive call of cat.
- Can be repaired using sized Types (Abel).
  - Using sized types one can apply size preserving or increasing functions to corecursive calls.
  - The code in the following usually requires decorations by sized types in order to termination check.

Executable Program

main : NativeIO Unit
main = translateIOConsole cat

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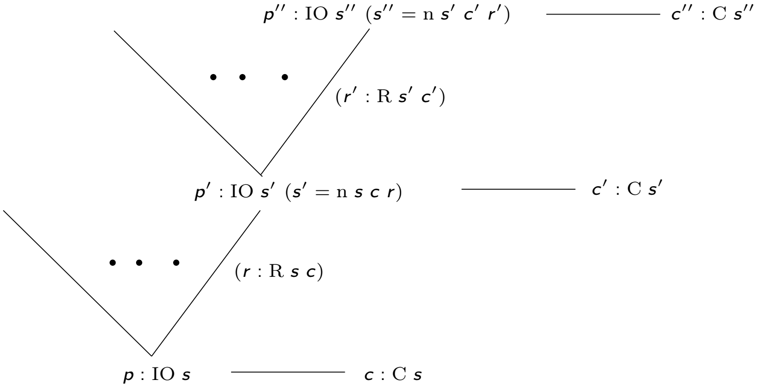
State Dependent IO-Trees



State Dependent IO – Interface

```
record IOInterfaceˢ : Set₂ where
  field
    Stateˢ    : Set₁
    Commandˢ  : Stateˢ → Set₁
    Responseˢ : (s : Stateˢ) → Commandˢ s → Set
    nextˢ     : (s : Stateˢ) → (c : Commandˢ s)
                → Responseˢ s c
                → Stateˢ
```

State Dependent IO

```
record IOˢ (A : S → Set) (s : S) : Set₁ where
    coinductive
    field
      forceˢ : IOˢ' A s

data IOˢ' (A : S → Set) : S → Set₁ where
   doˢ' :      {s : S} → (c : C s)
           → ( f : (r : R s c) → IOˢ A (next s c r) )
           → IOˢ' A s
   returnˢ' : {s : S} → (a : A s) → IOˢ' A s
```

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