Closing the Validation Gap or Verifying Railway Interlockings in Agda

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Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety

Proviso

- Background in mathematical logic, proof theory and type theory.
- Be prepared of misuse or naive use of terminology from software engineering.

Examples of Validation Problems

Closing the Validation Gap

Case Study: Formalisation of Railway Interlocking System

Proof of Safety

Exam Question

- Assume you have two planes:
  - The code for the first one has been **fully verified** using automated and interactive **theorem proving**, but the plane has not been tested.
  - The code for the second one has not been verified this way, but the plane has been **thoroughly tested**.
- Which one do you choose to use?

Validation Gap

- **Verification** can be done in a machine checked way.
- Verification is only relative to a given specification.
- How do you know that the specification guarantees that the program fulfils the requirements?
- **Validation** checks that a program fulfils the requirements or a specification guarantees that the requirements are fulfilled.
  - Cannot be done formally.

Example Incomplete Specification

- We have written a program for controlling a railway interlocking system using SPARK Ada.
- Specification based on Hoare logic (pre and post conditions).
- **Verification** was carried out in a **machine checked** way.
- When running the program it was incorrect.
  - **Trains disappeared**.
  - Forgotten to add to the specification that trains should not get lost.
  - This happened in real world as well (disappearance of trains from a US control system of railways).

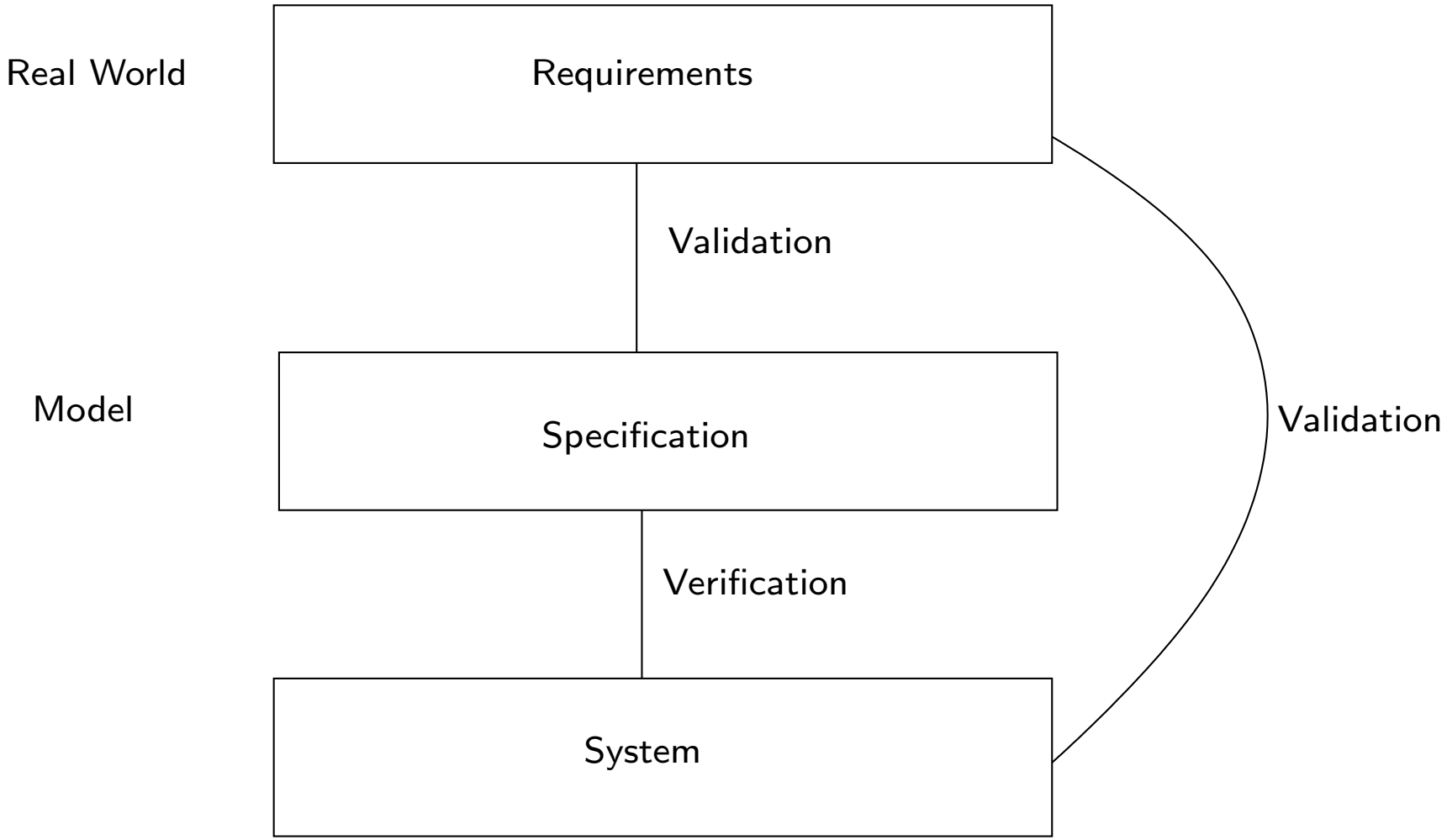
Complexity of Specification

- Tobias Nipkow has verified the security of a hotel key system.
- **Specification was substantially longer than the program**.
- Maybe it is easier to see that the program is secure than that the specification guarantees security?

Closing the Validation Gap

- **Verification** can be done **provably correct** or using systematic thorough testing.
  - We can guarantee (up to a certain degree).
- **Validation** can only be done using semi-formal, systematic methods.
  - We cannot guarantee it.
- We cannot avoid a **gap** between specification and requirements.
- However we can **make the gap as small as possible**.

Requirements - Specification - System



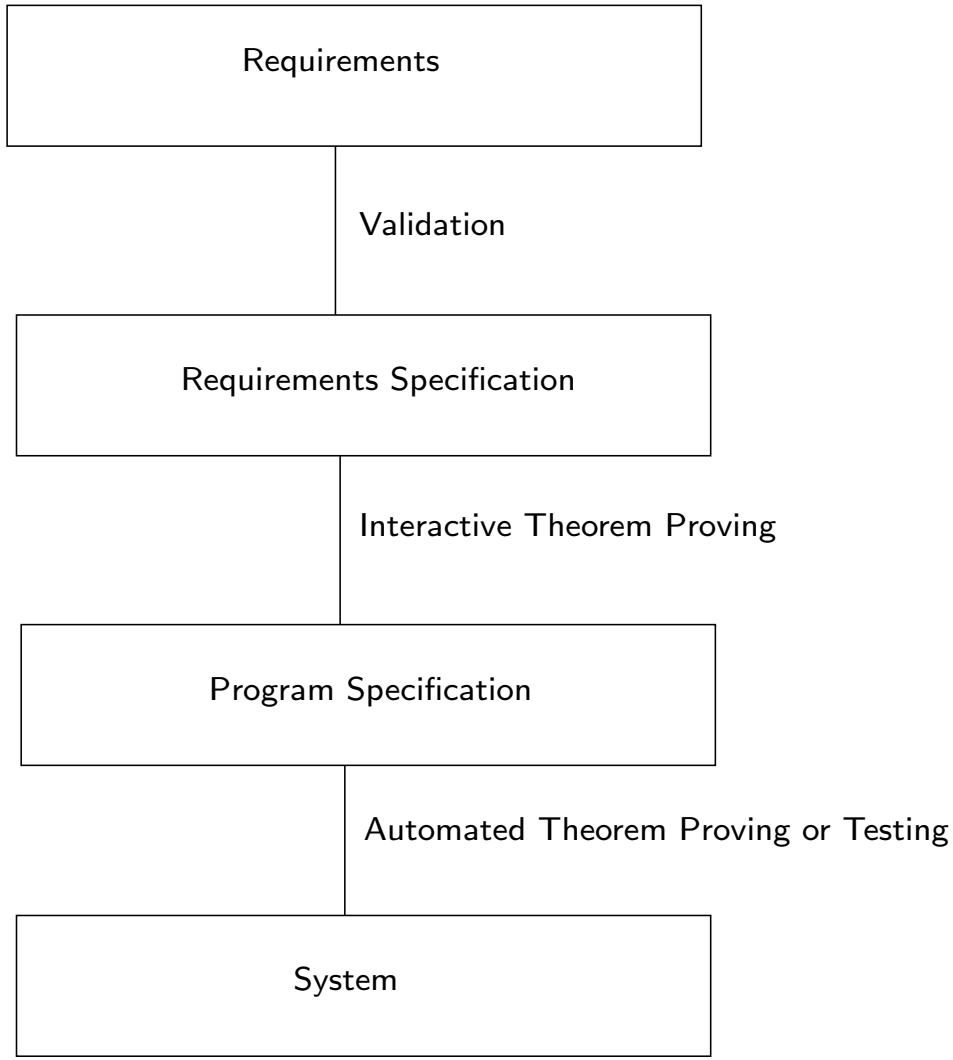
Suggestion to have two Specifications

- **Requirements specification** which is as close as possible to the requirements.
  - Corresponds as close as possible to a model of the real world situation.
  - Example: In railway interlocking systems model of railways.
- **Program specification** which is used to verify the program.
  - Should make it easy to verify that a program fulfils the specification.
  - Example: In railway interlocking systems signalling principles
    E.g.: If signal A is green, signal B is red.

Interactive vs Automated Theorem Proving

- That the **program fulfils the program specification** is typically provable by **automated theorem proving**.
  - In case of railway interlocking systems show that a railway interlocking system fulfils signalling principles.
- That the **program specification implies the requirements specification** is typically provable by **interactive theorem proving**.

Requirements and Program Specification

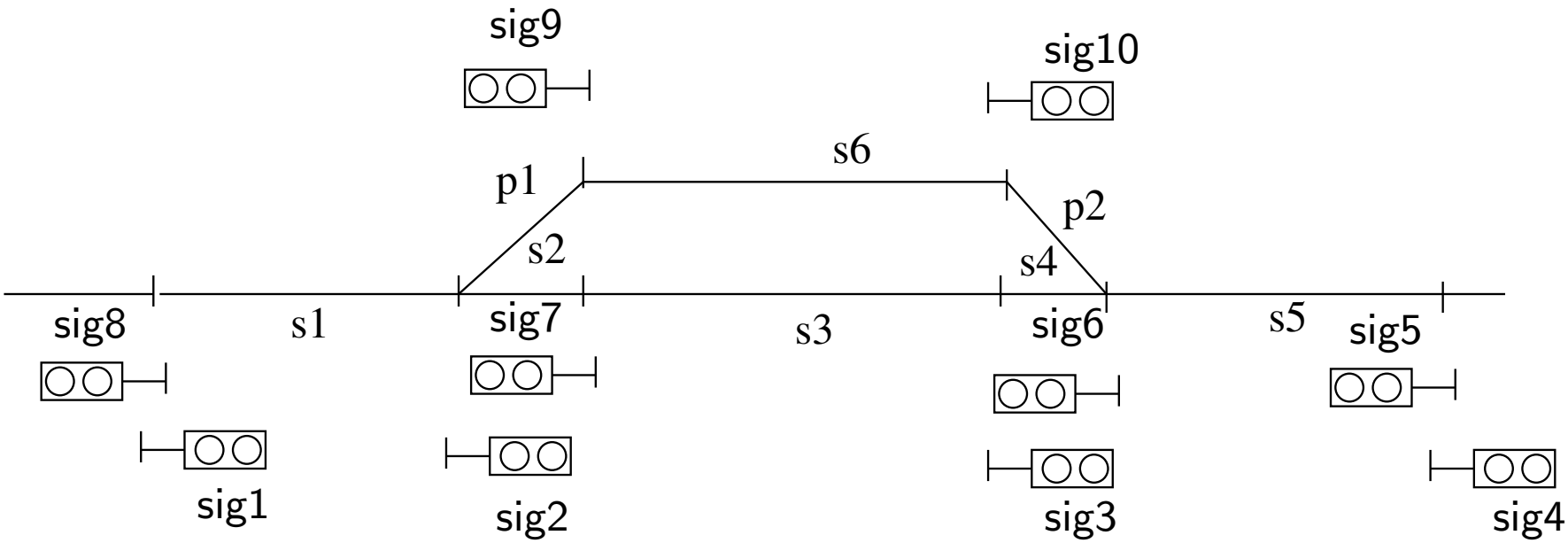


Track Segments

- The basic unit into which one divides a rail yard is that of a **track segment**.
- A track segment is stretch of a track without any further smaller parts, which are significant for an analysis of a interlocking system.
  - there are no sets of points in between (but a set of points might form one segment)
  - there are no crossings in between,
  - they are not divided by signals into parts.

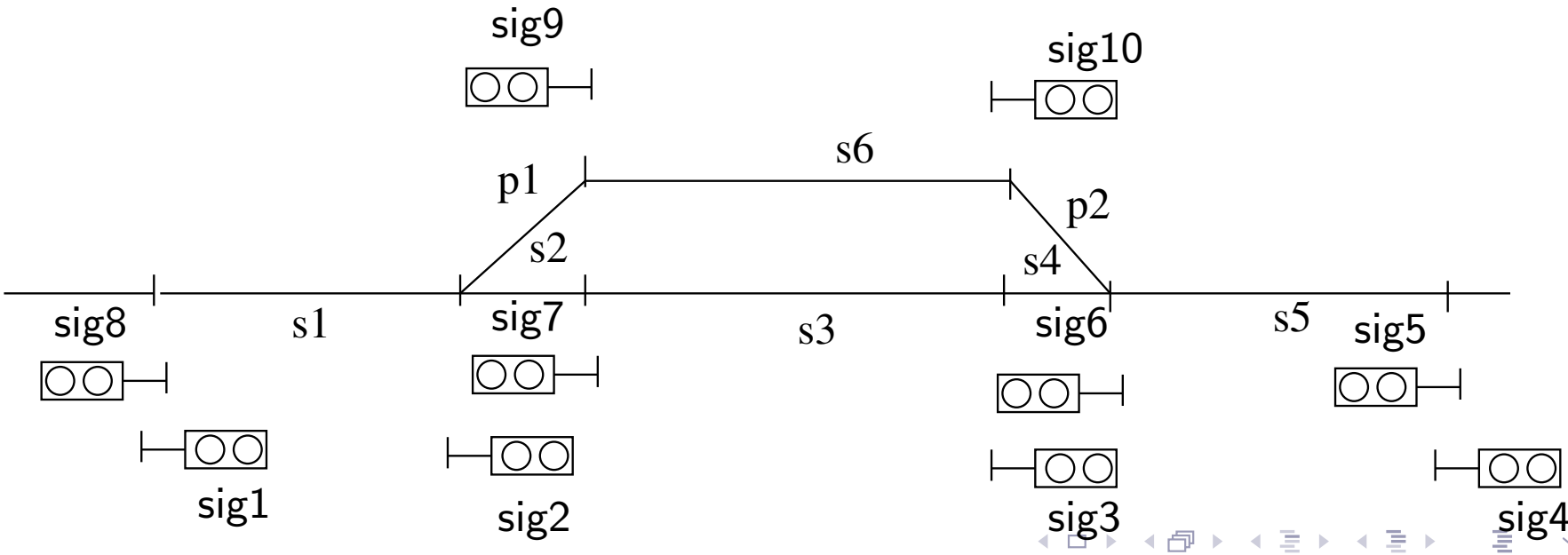
Example

- In the following example we have track segments s1 - s6.
- The two branches of the set of points p1 form segment s2.
- The two branches of the set of points p2 form segment s4.



Signals

- Signals control the access from one train segment to the next one.
- They are drawn in the direction of use, e.g. Signal sig2 is visible from s1 and controls access to s2.
- In the example sig2, sig7, sig9, control access to the set of points p1, and sig3, sig6, sig10 control access to p2.
- sig1, sig5 control access to s1, s5 respectively, and sig8, sig4 control access to the neighbouring rail yards.

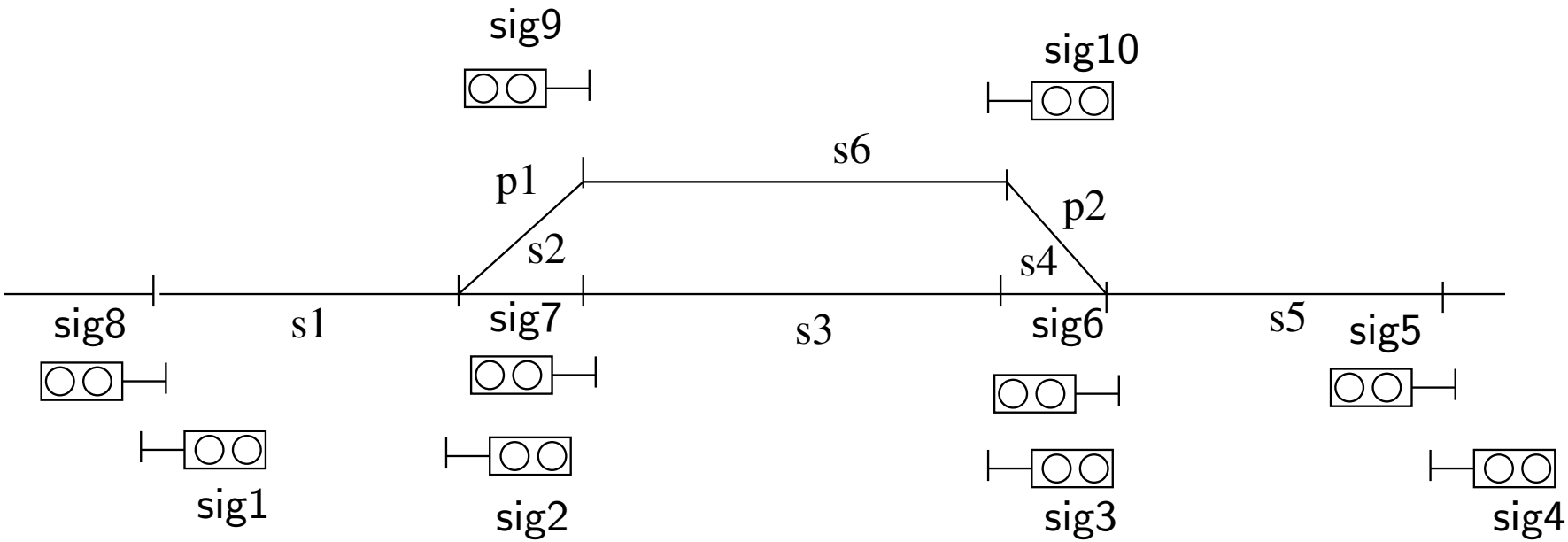


Train Routes

- The control system for such a rail yard has several **train routes**.
  - A **train route** is a sequence of track segments, the train can follow without ever having to stop in between (except in emergency cases).
  - The beginning of a train route and its end should be delimited by signals.
    - The first one prevents entering the train route, the second one, delimits access from this train route to the following train routes.
  - The segment before the guarding signal belongs to the route.

Train Routes

- So we have a train route (s1,s2,s6)
  - with segments s1,s2, s6
  - guarded by signal sig2
- Routes r1, r2 are connected if after having traversed route r1 one can proceed to route r2
  - route (s1,s2,s6) and route (s6,s4,s5) are connected.



Formalisation in Agda

- We follow
  Karim Kanso and Anton Setzer: *A light-weight integration of automated and interactive theorem proving*. Mathematical Structures in Computer Science, FirstView, 2014, pp. 1 - 25.

Formalisation

- We have sets and relations

$$
\begin{array}{lcl}
\text{Segment} & : & \text{Set} \\
\text{Train} & : & \text{Set} \\
\text{Route} & : & \text{Set} \\
\text{Connected} & : & \text{Route} \to \text{Route} \to \text{Set} \\
\text{SegInRoute} & : & \text{Segment} \to \text{Route} \to \text{Set}
\end{array}
$$

Model

- Time is given as

$$\mathrm{Time} = \mathbb{N} : \mathrm{Set}$$

- Depending on $t : \mathrm{Time}$ we assume

$$\begin{array}{lcl} \mathrm{trainRoute}_t & : & \mathrm{Train} \to \mathrm{Route} \\ \mathrm{signalAspect}_t & : & \mathrm{Route} \to \{\mathrm{proceed}, \mathrm{danger}\} \end{array}$$

Abstract Assumptions about Routes and Trains

- Single-Entry-Point: If two routes $route_1$ and $route_2$ are connected to route $route_3$, there is a segment (the one before the signal of $route_3$ which is in $route_1$ and $route_2$):

$$\begin{aligned}\forall route_1, route_2, route_3.&\text{Connected } route_1\ route_3\\ &\rightarrow\ \text{Connected } route_2\ route_3\\ &\rightarrow\ \exists segment.(\text{SegInRoute } segment\ route_1\\ &\qquad\wedge\text{SegInRoute } segment\ route_2)\end{aligned}$$

- Trains follow connected routes and obey signals:

$$\begin{aligned}\forall t, train.&(\text{trainRoute}_t\ train \equiv \text{trainRoute}_{t+1}\ train)\\ &\vee(\text{Connected } (\text{trainRoute}_t\ train)\ (\text{trainRoute}_{t+1}\ train)\\ &\wedge\text{signalAspect}_t(\text{trainRoute}_{t+1}\ train) \equiv \text{proceed})\end{aligned}$$

Abstract Signal Principle 1: Opposing Signals are not both Green

- If a segment is in two different routes, the signal of one of the routes must have aspect danger:

$$\begin{aligned}
&\forall t, \mathit{route}_1, \mathit{route}_2, \mathit{segment}.\\
&\quad \mathit{route}_1 \not\equiv \mathit{route}_2\\
&\quad \rightarrow \mathrm{SegInRoute}\ \mathit{segment}\ \mathit{route}_1\\
&\quad \rightarrow \mathrm{SegInRoute}\ \mathit{segment}\ \mathit{route}_2\\
&\quad \rightarrow (\mathrm{signalAspect}_t\ \mathit{route}_1 \equiv \mathit{danger}\\
&\qquad \vee \mathrm{signalAspect}_t\ \mathit{route}_2 \equiv \mathit{danger})
\end{aligned}$$

Abstract Signal Principle 2: Routes of Trains are Guarded

- If a train is using a route, all routes with access to the segments of this route are guarded by red signal:

$$\begin{aligned}&\forall t, \mathit{train}, \mathit{segment}, \mathit{route}.\\ &\quad \mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_t\ \mathit{train})\\ &\quad \rightarrow \mathrm{SegInRoute}\ \mathit{segment}\ \mathit{route}\\ &\quad \rightarrow \mathrm{signalAspect}_t\ \mathit{route} \equiv \mathrm{danger}\end{aligned}$$

Initial Condition

$$\forall \mathit{train}_1, \mathit{train}_2, \mathit{segment}$$
$$\mathit{train}_1 \not\equiv \mathit{train}_2$$
$$\rightarrow \neg(\mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_0\ \mathit{train}_1)$$
$$\wedge \mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_0\ \mathit{train}_2))$$

Collision Free

Theorem

Assume the above abstract conditions. Then trains don't collide, i.e.

$$\begin{aligned}&\forall t, \mathit{train}_1, \mathit{train}_2, \mathit{segment}\\ &\quad \mathit{train}_1 \not\equiv \mathit{train}_2\\ &\quad \rightarrow \neg(\mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_t\ \mathit{train}_1)\\ &\qquad \wedge \mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_t\ \mathit{train}_2))\end{aligned}$$

Proof of Theorem

- Induction on $t : \mathrm{Time}$.
- $t = 0$ follows by the initial condition.
- For $t \to t + 1$ assume $\mathit{train}_1, \mathit{train}_2, \mathit{segment}$ s.t.

$$\begin{aligned} &\mathit{train}_1 \neq \mathit{train}_2 \\ &\mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_{t+1}\ \mathit{train}_1) \\ &\mathrm{SegInRoute}\ \mathit{segment}\ (\mathrm{trainRoute}_{t+1}\ \mathit{train}_2) \end{aligned}$$

  and show a contradiction.
  - If none of the trains have moved (so their routes are as before) this follows by IH.

Proof of Theorem

- If only $train_1$ has moved we have:

$\mathrm{signalAspect}_t(\mathrm{trainRoute}_{t+1}\ train_1) \equiv proceed$
(since $train_1$ obeys signals)
$\mathrm{SegInRoute}\ segment\ (\mathrm{trainRoute}_{t+1}\ train_2)$
$\mathrm{SegInRoute}\ segment\ (\mathrm{trainRoute}_t\ train_2)$
(by $\mathrm{trainRoute}_{t+1}\ train_2 = \mathrm{trainRoute}_t\ train_2$)
$\mathrm{SegInRoute}\ segment\ (\mathrm{trainRoute}_{t+1}\ train_1)$
$\mathrm{signalAspect}_t\ (\mathrm{trainRoute}_{t+1}\ train_1) \equiv \mathrm{danger}$
(by Abstract Signal Principle 2)
Contradiction

- The case where only $train_2$ has moved follows similarly.

Proof of Theorem

- If both $train_1$, $train_2$ have moved to the same route we have

$\text{trainRoute}_{t+1}\ train_1 \equiv \text{trainRoute}_{t+1}\ train_2$
$\text{Connected}\ (\text{trainRoute}_t\ train_1)\ (\text{trainRoute}_{t+1}\ train_1)$
$\text{Connected}\ (\text{trainRoute}_t\ train_2)\ (\text{trainRoute}_{t+1}\ train_2)$
Since train routes of trains are connected
$\exists segment$
$\text{SegInRoute}\ segment\ (\text{trainRoute}_t\ train_1)$
$\wedge \text{SegInRoute}\ segment\ (\text{trainRoute}_t\ train_2)$
(by single entry to routes)
Contradiction to IH

Proof of Theorem

- If both $train_1$, $train_2$ have moved to different routes we have

$$\begin{aligned}
&\text{signalAspect}_t\ (\text{trainRoute}_{t+1}\ train_1) \equiv \text{proceed}\\
&\text{signalAspect}_t\ (\text{trainRoute}_{t+1}\ train_2) \equiv \text{proceed}\\
&\quad \text{(Since trains obey signals)}\\
&\text{trainRoute}_{t+1}\ train_1 \not\equiv \text{trainRoute}_{t+1}\ train_2\\
&\text{SegInRoute}\ segment\ (\text{trainRoute}_{t+1}\ train_1)\\
&\text{SegInRoute}\ segment\ (\text{trainRoute}_{t+1}\ train_2)\\
&(\text{signalAspect}_t\ (\text{trainRoute}_{t+1}\ train_1) \equiv \text{danger}\\
&\ \vee \text{signalAspect}_t\ (\text{trainRoute}_{t+1}\ train_2) \equiv \text{danger})\\
&\text{Contradiction}
\end{aligned}$$

Points in Routes of Trains are Locked

- Similarly we were able to show that under additional conditions on points we have
  If a set of points is in facing direction of a route of a train, then the set of points is locked.

Sketch of Reduction to Real Interlockings

- The conditions on $\mathrm{trainRoute}_t$ and $\mathrm{signalAspect}_t$ are still abstract.
- In order to reduce it to concrete interlockings we take the following steps
  - Formalise state (consisting of interlocking state, location circuits, trains).
  - Formalise desired inputs to state.
  - Define initial state.
  - Define functions computing next state depending on state and desired input.
  - Define concrete signalling principles and conditions on locations/trains for initial state and next state.
  - Show that the functions above fulfil these concrete conditions.
  - Compute $\mathrm{trainRoute}_t$, $\mathrm{signalAspect}_t$.
  - Show that concrete conditions above imply the abstract conditions on $\mathrm{trainRoute}_t$, $\mathrm{signalAspect}_t$.
  - Therefore the interlocking system is safe.

Evaluation

- Even in this simplified situation it is rather complicated to see that the signalling principles imply safety.
- In usual validation this is done by hand.
- In the above approach we have formalised it mathematically in Agda and shown that the signalling principles imply safety.
- Therefore the validation gap has been narrowed.

Conclusion

- **Validation gap** between Specification and Requirements.
- By having a **requirements specification** which is as close as possible to the requirements this gap can be narrowed.
- Replaces arguments which are carried out informally in the head of the validator by robust mathematical arguments.
- Two step verification:
  - **Step 1**: Program fulfils program specification.
  - **Step 2**: Program specification implies requirements specification.
- Full verification of real world interlocking system in Agda has been carried out (PhD thesis Karim Kanso).
  Interlocking system could be executed in Agda as an interactive program.

References

- Karim Kanso and Anton Setzer: *A light-weight integration of automated and interactive theorem proving*. Mathematical Structures in Computer Science, FirstView, 2014, pp. 1 - 25.
- Karim Kanso: Agda as a Platform for the Development of Verified Railway Interlocking Systems. PhD thesis, Department of Computer Science, Swansea University, Swansea, UK. http://www.swan.ac.uk/∼csetzer/articlesFromOthers/index.html
- Karim Kanso: Formal Verification of Ladder Logic. MRes thesis, Department of Computer Science, Swansea University, Swansea, UK. http://www.swan.ac.uk/∼csetzer/articlesFromOthers/index.html