Schemata for Proofs by Coinduction

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Proofs by induction are carried out by following schemata for induction, which makes it easier to carry out such kind of proofs than by using directly the fact that the natural numbers is the least set closed under zero and successor. So for proving $\forall x.\varphi(x)$, one doesn't define first $A := \{x \in \mathbb{N} \mid \varphi(x)\}$ and show that A is closed under 0 and successor. Instead, one uses the schema of induction. Although using the schema of induction amounts to essentially the same as showing the closure properties of A, using the schema of induction is much easier to use and to teach.

Proofs by coinduction usually follow directly the principle that the coinductively defined set is the largest set satisfying the principles of the coinductively defined set. For instance for carrying out proofs of bisimulation, one usually introduces a relation and shows that it is a bisimulation relation. This makes proofs by coinduction cumbersome and difficult to teach.

In this talk we will introduce schemata for coinduction which are similar to the schemata for induction. The use of the coinduction hypothesis is made easier by defining coinductively defined sets as largest sets allowing observations, rather than as largest sets closed under introduction rules. For instance the set Stream of streams of natural numbers is the largest set allowing observations $\mathrm{head} : \mathrm{Stream} \to \mathbb{N}$ and $\mathrm{tail} : \mathrm{Stream} \to \mathrm{Stream}$, rather than being the largest set closed under $\mathrm{cons} : \mathbb{N} \to \mathrm{Stream} \to \mathrm{Stream}$.

Based on this we will first introduce schemata for defining functions by primitive corecursion or guarded recursion. This is dual to the principle of primitive recursion. Then we define schemata for coinductive proofs of equality. Finally we introduce schemata for coinductively defined relations such as bisimulation relations.

We will give examples of how to carry out coinductive proofs on paper. These proofs will make use of the coinduction hypothesis, where restrictions that are dual to those for the use of the induction hypothesis in inductive proofs are used.

The general theory of schemata for coinductive proofs can be found in our article [1].

References

[1] Anton Setzer. How to reason coinductively informally. Accepted for publication in Reinhard Kahle, Thomas Strahm, Thomas Studer (Eds.): Festschrift on occasion of Gerhard Jäger's 60th birthday, Springer., 20 November 2015.