The Role of the Coinduction Hypothesis in Coinductive Proofs

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When carrying out proofs by induction, we follow schemata of inductive proofs rather than arguing that the set of natural numbers is the least set closed under 0 and successor. So for proving $\forall x.\varphi(x)$, we usually don't define $A := \{x \in \mathbb{N} \mid \varphi(x)\}$ and show that it is closed under 0 and successor, but use the principle of induction. Although using A and the principle of induction amount essentially to the same, using the induction principle is much easier to use and to teach.

When carrying out proofs by coinduction, one usually argues similarly as to proving the closure of the set A as above. For instance for carrying out proofs of bisimulation, one usually introduces a relation and shows that it is a bisimulation relation. This makes proofs by coinduction cumbersome and difficult to teach.

In this talk we develop schemata for reasoning coinductively which are similar to those used for inductive proofs. Proofs will make use of the coinduction hypothesis, where certain restrictions on its use need to be observed.

The use of the coinduction hypothesis is facilitated by the fact that we will define coinductive sets not as largest sets closed under constructors, i.e. by their introduction rules, but as largest sets allowing observations, i.e. by their elimination rules. For instance the set Stream of streams of natural numbers is the largest set allowing observations $\text{head} : \text{Stream} \to \mathbb{N}$ and $\text{tail} : \text{Stream} \to \text{Stream}$.

The schemata we introduce will be schemata for corecursive definitions of functions, for coinductive proofs of equalities on coinductively defined sets, and coinductive proofs of coinductively defined relations. A special case of the latter is the proof of bisimulation on labelled transition systems. We will give examples on how to carry out such kind of coinductive proofs.