▶ ANTON SETZER, *The extended predicative Mahlo Universe in Explicit Mathematics – model construction.*
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This is joint work with Reinhard Kahle, Lisbon. In [3] Setzer introduced the Mahlo universe V in type theory and determined its proof theoretic strength. This universe has a constructor, which depends on the totality of functions from families of sets in the universe into itself. Essentially for every such function f a subuniverse U_f of V was introduced, which is closed under f and represented in V. Because of the dependency on the totality of functions, not all type theoretists agree that this is a valid principle, if one takes Martin-Löf Type Theory as a foundation of mathematics.

Feferman's theory of Explicit Mathematics [1] is a different framework for constructive mathematics, in which we have direct access to the set of partial functions. In such a setting, we can avoid the reference to the totality of functions on V. Instead, we can take arbitrary partial functions f, and try to form a subuniverse U_f closed under f. If f is total on U_f, then we add a code for it to V. In [2] we developed a universe based on this idea (using m as a name for V and sub as a name for U), and showed that we can embed the axiomatic Mahlo universe, an adaption of the Mahlo construction as in [3] to Explicit Mathematics, into this universe. We added as well an induction principle, expressing that the Mahlo universe is the least one. Since the addition of U_f to V depends only on elements of V present before U_f was added to V, it can be regarded as being predicative, and we called it therefore the extended predicative Mahlo universe.

In this talk we construct a model of the extended predicative Mahlo universe in a suitable extension of Kripke-Platek set theory, in order to determine an upper bound for its proof theoretic strength. The model construction adds only elements to the Mahlo universe which are justified by its introduction rules. The model makes use of a new monotonicity condition on family sets, the notion of a monotone operator for defining universes, and a special condition for closure operators. This is an alternative to Richter's $[\Gamma, \Gamma']$ operator for defining closure operators.

[1] SOLOMON FEFERMAN ***Algebra and Logic***, (John Crossley, editor), Springer, 1975, pp. 87–139.

[2] REINHARD KAHLE AND ANTON SETZER, *An extended predicative definition of the Mahlo universe*, ***Ways of proof theory***, (Ralf Schindler, editor), Ontos Series in Mathematical Logic, Ontos Verlag, Frankfurt (Main), Germany, 2010, pp. 315–340.

[3] ANTON SETZER, *Extending Martin-Löf Type Theory by one Mahlo-Universe*, ***Archive for Mathematical Logic***, vol. 39 (2000), pp. 155–181.