

# CSCM10 Research Methodology Specification and Verification

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[http://www.cs.swan.ac.uk/~csetzer/lectures/  
computerScienceProjectResearchMethods/current/index.html](http://www.cs.swan.ac.uk/~csetzer/lectures/computerScienceProjectResearchMethods/current/index.html)

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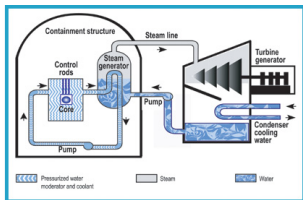
- ① Critical Systems
- ② Specification
- ③ Verification
- ④ Dependent Type Theory
- ⑤ Security
- ⑥ Theoretical Topics

- 1 Critical Systems
- 2 Specification
- 3 Verification
- 4 Dependent Type Theory
- 5 Security
- 6 Theoretical Topics

**Definition:** A critical system is a

- computer, electronic or electromechanical system
- the failure of which may have serious consequences, such as
  - substantial financial losses,
  - substantial environmental damage,
  - injuries or death of human beings.

# Example 1: Nuclear Power

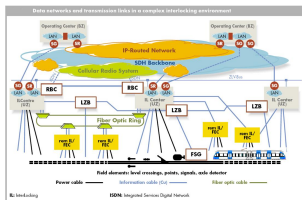


# Example: Medical Devices





# Example: Railways





# Failure of a Critical System



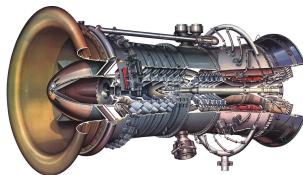
# Failure of a Critical System



# Industrial Partners of Swansea Group in Safe and Secure Systems

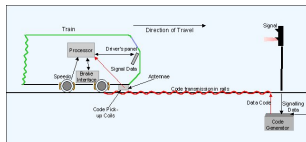
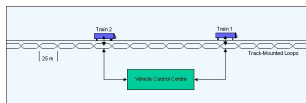


**SIEMENS**



- The department of Computer Science has a big group working on logic, theoretical computer science and applications to verification of software and hardware.
- Long experience in working with verification of software and hardware.
- Industrial connections with companies such as Rolls Royce, Developers of Electronic Payment Systems, Siemens.

- Well established collaboration with Siemens Rail Automation (Chippenham, formerly Invensys RailSystems) on modelling and verification of new generations of railway interlocking systems.
  - Currently working on radio controlled moving block systems (ERTMS).



- Verification in the Railway Domain
  - Ulrich Berger
  - Phil James
  - Faron Moller
  - Liam O'Reilly
  - Markus Roggenbach
  - Monika Seisenberger
  - Anton Setzer
- Embedded Systems and Testing
  - Arnold Beckmann,
  - Markus Roggenbach.

- ① Critical Systems
- ② **Specification**
- ③ Verification
- ④ Dependent Type Theory
- ⑤ Security
- ⑥ Theoretical Topics

# Why Formal Specification?

- Natural language specification can be ambiguous.
  - “The output is a red light or a green light”.
    - Do you mean “either or” or “inclusive or”?



# Why Formal Specification?

- Formal specification enforce precision.
  - Example: If the level of the water in the tank is above a certain level, the plug valve must be closed.  
Do you mean
    - maximum level,
    - average,
    - medium,
    - or ... (lots of other possibilities)?

# Why Formal Specification?

- Natural language specifications don't allow formal verification.

# Challenges in Specification

- Finding a suitable language which is
  - expressive
  - and simple enough for the user to understand it.
- Describe the meaning of specification languages (semantics).
- For specifying a formal system, determine the right
  - notions,
  - level of abstraction

# Example

- Distant signals and main signal in railways.  
Is
  - the main signal a function of the distant signal,
  - or the distant signal a function of the main signal,
  - or are main signal and distant signal in a relation.
- During specification, often need to switch between different choices.
- General problem of modelling systems.

- Algebraic Specification.
  - Markus Roggenbach (CASL)
  - John Tucker (theory of algebraic specification)
- Process Algebras
  - Faron Moller (CCS),
  - Markus Roggenbach (CSP-CASL),
  - Anton Setzer (CSP-Agda).

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# Verification

- Verification is the process of determining whether a software product coincides with its specification.
- Many methods.
- Main method is testing.
- Testing usually not complete.
- In order to guarantee that a program is guaranteed to be correct, one needs prove that the output of software coincides with the specification.
  - Necessary especially for critical systems.
  - Increasingly used for general systems, e.g. by Microsoft, to guarantee security of its software.
- Done using theorem proving techniques.

## 1. Theorem proving by hand.

- What mathematicians do all the time.
- Will remain in the near future the main way for proving theorems.
- Problem: Errors.
  - As in programs after a certain amount of lines there is a bug, after a certain amount of lines a proof has a bug.
  - The problem can only be reduced by careful proof checking, but not eliminated completely.
- Unsuitable for verifying large software and hardware systems.
  - Data usually too large.
  - Likely that one makes the same mistakes as in the software.



## 2. Theorem proving with some machine support.

- Machine checks the syntax of the statements, creates a good layout, translates it into different languages.
- Theorem proving still to be done by hand.
- **Example:** most systems for specification of software.
- **Advantages:**
  - Less errors.
  - User is forced to obey a certain syntax.
  - Specifications can be exchanged more easily.
- **Disadvantage:** Similar to 1.

## 3. Interactive Theorem Proving.

- Proofs are fully checked by the system.
- Proof steps have to be carried out by the user.
- **Advantages:**
  - Correctness guaranteed (provided the theorem prover is correct).
  - Everything which can be proved by hand, should be possible to be proved in such systems.

## 4 Ways of Proving Theorems

- (Interactive theorem proving)
  - **Disadvantages:**
    - It takes much longer than proving by hand.
    - Similar to programming:
      - To say in words what a program should do, doesn't take long.
      - To write the actual program, can take a long time, since much more details are involved than expected.
    - Requires experts in theorem proving.

## 4. Automated Theorem Proving.

- The theorem is shown by the machine.
- It is the task of the user to
  - state the theorem,
  - bring it into a form so that it can be solved,
  - usually adapt certain parameters so that the theorem proving solves the problem within reasonable amount of time.

# 4 Ways of Proving Theorems

- (Automated theorem proving)
  - **Advantages**
    - Less complicated to “feed the theorem into the machine” rather than actually proving it.  
Might be done by non-specialists.
    - Sometimes faster than interactive theorem proving.

# 4 Ways of Proving Theorems

- (Automated theorem proving)
  - **Disadvantages**
    - Many problems cannot be proved automatically.
    - Can often deal only with finite problems.
    - We can show the correctness of one particular processor.
    - But we cannot show a theorem, stating the correctness of a parametric unit (like a generic  $n$ -bit adder for arbitrary  $n$ ).
    - In some cases this can be overcome.
    - Limits on what can be done (some hardware problems can be verified as 32 bit versions, but not as 64 bit versions).

# Verification in Industry

- Most verification done using testing.
- Some theorem proving by hand and with some machine support done.
- Increasingly theorem proving using automated theorem proving done.
  - Investment of Microsoft in various automated theorem provers.
  - Package management in Linux became much faster due to use of SAT solvers (Automated Theorem Provers).

- Interactive theorem proving on its way into industry.
  - Typical scenario:
  - General properties of a system proved using interactive theorem proving
    - E.g. signalling principles formally expressed safety.
  - That a concrete installation is in accordance with those general principles done using automated theorem proving.
    - E.g. show that a railway interlocking system fulfils signalling principles.



- Verification using automated theorem provers (ATP).
  - Oliver Kullmann (SAT solvers, e.g. OK-Solver)
- Verification using interactive theorem provers (ITP).
  - Markus Roggenbach (Isabelle),
  - Ulrich Berger (Minlog, Coq),
  - Monika Seisenberger (Minlog),
  - Anton Setzer (Agda).

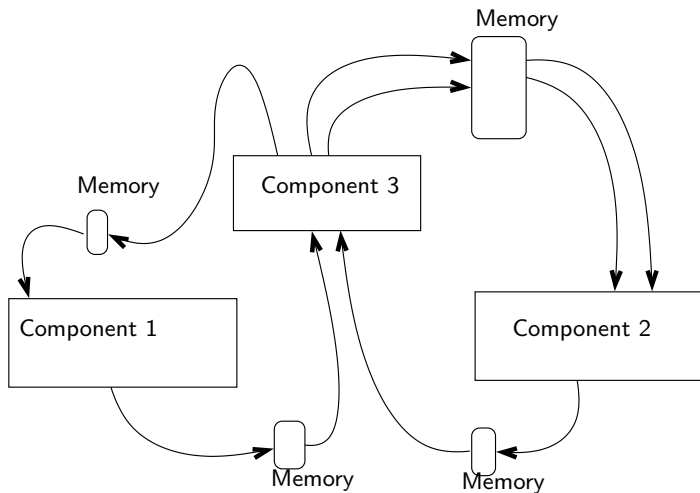
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- Agda is a theorem prover which is as well a prototype of a dependently typed programming language.
- In Agda proofs and programs are the same.
- A proof of a theorem  $A$  is a program  $p$  of type  $A$  written as

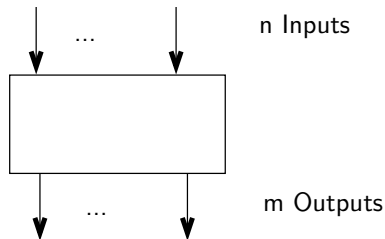
$$p : A$$

- Relatively easy for programmers, since they don't need to learn a different activity.
- Agda uses the novel concept of dependent types.
- In Swansea Anton Setzer is expert in Agda.

# Example: Boolean Circuits



# What is a Component?



A Boolean Component can be represented by a

$$f : \text{Bool}^n \rightarrow \text{Bool}^m$$

# What is the type $\text{Bool}^n \rightarrow \text{Bool}^m$ ?

- $\text{Bool}^n \rightarrow \text{Bool}^m$  is a type depending on  $n, m : \mathbb{N}$ .
- In most languages you don't have any dependent type. You need to replace this by  $\text{List}(\text{Bool}) \rightarrow \text{List}(\text{Bool})$ .
- In C++ you can define

$$\text{Bool}^n \rightarrow \text{Bool}^m$$

but only, if  $n, m$  are known at compile time.

- Disallows dynamic dependencies, e.g. depending on user input.
- In Agda we can directly use  $\text{Bool}^n \rightarrow \text{Bool}^m$  as a dependent type.

## Example 2: Grammars

- Assume you want to write programs which manipulate Java programs.
  - E.g. change a variable not using brute query replace.
- One way of doing this:
  - Define a data type of Java programs.
  - Translate strings into this data type and back again.
  - Write programs which work on this data type of Java programs.

## Example 2: Grammars

- An oversimplified grammar for Java might start as follows:

JavaProg             $\longrightarrow$  "class" identifier "{ " JavaProgBody " } "  
JavaProgBody       $\longrightarrow$  ( VariableDecl ) \* ( MethodDecl ) \*  
VariableDecl        $\longrightarrow$  TypeDecl VariableName ";"  
...



# Transformers of Java Programs

- Let Grammarsymbol be the set of terminals and non-terminals (JavaProg, JavaProgbody, ...).
- For each Grammarsymbol  $S$  we define the type  $\llbracket S \rrbracket$  of entities of this type, e.g.
  - $\llbracket \text{TypeDecl} \rrbracket = \text{String}$ .
  - $\llbracket \text{VariableName} \rrbracket = \text{String}$ .
  - $\llbracket \text{VariableDecl} \rrbracket = \text{String} \times \text{String}$ .
- $\llbracket S \rrbracket$  is a **dependent type** depending on  $S : \text{GrammarSymbol}$ .

# Type of the Parser

Parser : (GrammarSymbol  $\times$  String)  $\rightarrow$  Bool

Transformer : ( $S$  : GrammarSymbol)  
 $\rightarrow$  ( $s$  : String)  
 $\rightarrow$  Parser( $S, s$ ) == true  
 $\rightarrow$   $\llbracket S \rrbracket$

- Makes heavy use of the dependent type  $\llbracket S \rrbracket$ .
- Parser Libraries in C++, Haskell, Agda have been built based on this idea.

# Generative Programming

- These are examples of generative programming.
- In generative programming you want to build highly generic programs, which generate and manipulate programs from elements of data types.

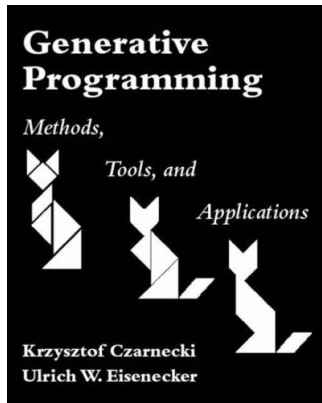
# Generative Programming

- So we have
  - a base data type `BaseType` (like `GrammarSymbol` before),
  - a type of programs `Program(S)` based on  $S : \text{BaseType}$  (like  $\llbracket S \rrbracket$  before),
  - operations which manipulate `Program(S)`, e.g.

$$\text{transform1} : ((S : \text{BaseType1}) \times \text{Program1}(S)) \rightarrow \text{BaseType2}$$
$$\text{transform2} : ((S : \text{BaseType1}) \times \text{Program1}(S)) \rightarrow \text{Program2}(\text{transform1}(S, s))$$

# Generative Programming

- Now we can create factories for generating programs.
- Replace handcrafted programs by generated programs.
- Similar to step from pre-industrial to industrial age.



# Dependent Types for Writing Verified Programs

- Assume we want to assign a type to a sorting function `sort` on lists of natural numbers.
- In most programming language, the type of it is essentially

$$\text{sort} : \text{NatList} \rightarrow \text{NatList}$$

for the type of lists of natural numbers `NatList`.

- In dependent type theory, we can demand more correctness, namely that its type is

$$\text{sort} : \text{NatList} \rightarrow \text{SortedList} .$$

- We assume some notion of `NatList` (list of natural numbers).

- What is SortedList?
  - An element of SortedList is a list which is sorted.
  - It is a pair  $\langle l, p \rangle$  s.t.
    - $l$  is a NatList.
    - $p$  is a proof or verification that  $l$  is sorted:
    - $p : \text{Sorted}(l)$ .

# Sorted Lists

- For the moment, ignore what is meant by  $\text{Sorted}(l)$  as a type.
- Only important:  $\text{Sorted}(l)$  depends on  $l$ .
  - $\text{Sorted}(l)$  is a predicate expressed as a type.
- Elements of  $\text{SortedList}$  are pairs  $\langle l, p \rangle$  s.t.
  - $l : \text{NatList}$ .
  - $p : \text{Sorted}(l)$ .
- $\text{Sorted}(l)$  is a dependent type.



## Sorted Lists (Cont.)

- An element of  $\text{Sorted}(l)$  will be a **proof** that  $l$  is sorted.
- If  $l$  **is sorted**, then  $\text{Sorted}(l)$  will be provable, and therefore **will have an element**.
  - It is possible to write a program which computes an element of  $\text{Sorted}(l)$ .
- If  $l$  is **not sorted**, then  $\text{Sorted}(l)$  will have no proof and it will therefore **no element**.
  - Then it is not possible to write a program which computes an element of  $\text{Sorted}(l)$ .

# The Dependent Product

- Then the pair  $\langle l, p \rangle$  will be an element of

$$\text{SortedList} := (l : \text{NatList}) \times \text{Sorted}(l) .$$

- SortedList is the type of pairs  $\langle l, p \rangle$  s.t.

- $l : \text{NatList}$ ,
- $p : \text{Sorted}(l)$ .

called the dependent product

- $\text{sort} : \text{NatList} \rightarrow ((l : \text{NatList}) \times \text{Sorted}(l))$  expresses:
  - sort converts lists into sorted lists.

# The Dependent Function Type

- From a sorting function we know more:
  - It takes a list and converts it into a sorted list **with the same elements**.
- Assume a type (or predicate)  $\text{EqElements}(l, l')$  standing for
  - $l$  and  $l'$  have the same elements.

# The Dependent Function Type

- A refined version of sort has type

$$(l : \text{NatList}) \rightarrow ((l' : \text{NatList}) \times \text{Sorted}(l') \times \text{EqElements}(l, l'))$$

- “ $\text{sort}(l)$  is a list, which is sorted and has the same elements”.
- “sort is a program, which takes a list and returns a sorted list with the same elements.”
- The type of sort is an instance of the **dependent function type**:
  - The result type depends on the arguments.

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- Cyberterrorism, General Security
  - Monika Seisenberger, Anton Setzer.
- Cryptocurrencies (Bitcoins, Blockchain).
  - Anton Setzer

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- Computability Theory and Limits of Computation
  - Ulrich Berger, Jens Blanck, Monika Seisenberger, John Tucker,
- Exact Real Number Computation
  - Ulrich Berger, Jens Blanck, Monika Seisenberger.
- Program Extraction
  - Ulrich Berger, Monika Seisenberger
- Proof Theory
  - Arnold Beckmann, Ulrich Berger, Monika Seisenberger, Anton Setzer, Jean Razafindrakoto.



- Complexity Theory
  - Arnold Beckmann, Oliver Kullmann, Faron Moller, Jean Razafindrakoto.
- Formal Argumentation
  - X. Fan.

# Conclusion

- Critical Systems require more formal specification and verification.
- Expertise in Swansea in specification and verification.
- Problems of natural language specification can be overcome by formal specification.
- Verification techniques – from proving by hand to interactive and automated theorem proving.
- Agda as an example of a programming language based on dependent types.
- Use of dependent types for generative programming.
- Research related to Security.
- Wide range of theoretical topics covered in Swansea.