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► ALEXANDER A. RAZBOROV, *Complexity of resolution proofs*.
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Complexity of propositional proofs plays as important a role in the theory of feasible proofs as the role played by the complexity of Boolean circuits in the theory of efficient computation. In many cases it provides a very elegant and combinatorially clean framework for studying provability of Σ_0^b-formulae in first-order theories of feasible arithmetic that bears essentially the same message as the original framework of first-order ("uniform") provability.

During several last years, many talks were delivered in various places on general aspects of either the whole discipline or its rather broad parts like Algebraic Proof Systems. It seems, by now the field has matured enough, and the time for more specialized survey talks has come. This talk is designed in such a way, and we choose for it one of the lowest levels in the hierarchy of propositional proof systems, Resolution. (Historically this was also the first p.p.s. considered by Tseitin in his seminal paper of 1968!)

Despite its simplicity, establishing lower bounds is a hard task not only for general resolution, but even for its more restricted versions like Regular Resolution. We survey known results in this direction, proved both by purely combinatorial arguments (like Haken's bottleneck technique) and by more general reductions to related problems in Complexity Theory (interpolation-like theorems).

Much of the research in the area is concentrated on the proof complexity of so-called *Pigeon-Hole-Principle*, and we will see how drastically it changes when we vary the number of pigeons (comparatively to the number of holes). We focus our attention on the central open problem in the area: understanding the proof complexity of this principle in case when the number of pigeons is very big, potentially infinite. We sketch some partial results and approaches toward this goal aimed at *regular* resolution. On our way, we recall a beautiful characterization of the latter system in purely computational terms of read-once branching programs for some specific search problems. The existence of such a characterization is something absolutely unique in the whole area of complexity of propositional proofs. We will see some related models of Rectangular Calculus and Transversal Calculus, and we will see some related (computational) bounds for read-once branching programs.

The last part of the talk consists of results obtained jointly with A. Wigderson and A. Yao.

► ANTON SETZER, *The role of large cardinals in ordinal notation systems*.
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Since Gentzen's analysis of Peano Arithmetic, one goal in proof theory has been the reduction of the consistency of mathematical theories to the well-ordering of ordinal notation systems. In the case of Gentzen's system and slight extensions the well-ordering of the systems is quite intuitive. Stronger ordinal notation systems are usually developed by using cardinals, large cardinals or their recursive analogues and the main intution is developed from set theory. Therefore they are no longer as intuitively well-ordered as the weaker systems, an obstacle for the understanding of such systems for non-specialists.

Ordinal systems is an alternative presentation of ordinal notation systems in such a way, that we have intuitive well-ordering arguments. They are defined in such a way that we can transfinitely enumerate all ordinal notations by repetitively selecting out of the set of ordinals, which are denoted by using ordinals previously chosen, the least element not chosen before with respect to some (well-ordered) termination ordering. In order to guarantee the correctness of this process and that it enumerates all ordinals, the following conditions are required:

- An ordinal notation is finite and refers only to smaller ordinals.
- If $\alpha < \beta$ can be denoted, then α is below some of the ordinals, β is denoted from, or the denotation of α is with respect to the termination ordering less than the denotation of β.
- If A is a set of ordinal notations which is well-ordered, the set of ordinals which can be denoted from ordinals in A is well-ordered with respect to the termination ordering.

An ordinal system is elementary, if the above condition can be verified in primitive recursive arithmetic, and elementary ordinal systems reach all ordinals below the Bachmann-Howard ordinal. In order to get beyond this bound, the analogue of cardinals in this approach is needed.

It turns out that what is needed are subprocesses: Instead of choosing in the main process the next ordinal directly, at every stage we need to start a subprocess in an ordinal system, which is relativized with respect to the ordinals already denoted in the main process. Once this subprocess is complete and has enumerated all ordinals in the relativized system, we can verify that the set of ordinals denotable from the ones previously selected in the main system is well-ordered with respect to the termination ordering and we can therefore select the next ordinal. For stronger ordinal notation systems, a more complicated arrangement of such subprocesses is necessary and therefore the role of cardinals in ordinal notation systems becomes clear: they are a way of organizing these processes.

We will analyze the relationship between these processes and cardinals in the case of regular cardinals, inaccessible cardinals and Mahlo cardinals.

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The foundational program to study the principles and ordinals which are implicit in a predicative conception of the universe of sets of natural numbers led to the progression of systems of ramified analysis up to the famous Feferman-Schütte ordinal Γ_0 in the early sixties. Since then numerous theories have been found which are not prima facie predicatively justifiable, but nevertheless have predicative strength in the sense that Γ_0 is an upper bound to their proof-theoretic ordinal. It is common to all these predicative theories that their analysis requires methods from predicative proof theory only, in contrast to the present proof-theoretic treatment of stronger impredicative systems. On the other hand, it has been well-known for a long time that there are natural systems which have proof-theoretic ordinal greater than Γ_0 and whose analysis makes use just as well of methods which every proof-theorist would consider to be predicative. Nevertheless, not many theories of the latter kind have been known until recently.

Metapredicativity is a new general term in proof theory which describes the analysis and study of formal systems whose proof-theoretic strength is beyond the Feferman-Schütte ordinal Γ_0, but which are nevertheless amenable to *predicative methods*. It has turned out only recently that the world of metapredicativity is extremely rich and that it includes many natural and foundationally interesting formal systems.

In this talk we give a general survey and introduction to metapredicativity. In particular, we discuss various examples of metapredicative systems, including (i) subsystems of second order arithmetic, (ii) first and second order fixed point theories, (iii) extensions of Kripke-Platek set theory without foundation by reflection principles, and (iv) systems of explicit mathematics with universes.

Relevant keywords for our talk are: arithmetical transfinite recursion and dependent