propositional logic do not even have failing deduction attempts. Usually information regarding nonprovability of a sequent has therefore to be extracted by a kind of metaargumentation from an unending sequence of inference steps. (For instance by considering loops in this sequence.) This situation also shows in the usual semantical completeness proofs for intuitionistic logic as may be found in the monographs. Such completeness proofs are much more complicated than similar proofs for classical logic. We are going to show that these problems may be solved by considering so called bicomplete sequent calculi for intuitionistic propositional logic. By this we mean calculi by which any sequent not provable in intuitionistic logic is refutable in a sense to be made precise. We are going to show that failing deduction attempts in such calculi provide a very perspicuous means for obtaining semantical counterexamples for arbitrary nonprovable formulae of intuitionistic logic. Moreover these calculi also yield a thorough analysis of the concept of a semantics itself: To any such calculus there is a canonical semantics based on it, and the relation between any bicomplete calculus and its semantics exactly mirrors the relation between classical semantics and the usual tableaux calculi for classical provability. Therefore in a sense these calculi serve to embed semantical approaches into proof theory and thus provide a means to make semantical approaches fruitful for automated deduction in intuitionistic logic.

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In the article "Well-ordering proofs in Martin-Löf's Type Theory" (submitted) we introduced the notion of ordinal notation systems "from below". These were systems of terms OT built from one ordinal function f (which codes several ordinal functions usually needed) together with an ordering $<$ such that for some additional ordering $<'$ on the arguments of f essentially the following conditions hold:

- f is inflating, i.e., $\beta_i < f(\vec{\beta})$.
- The well-foundedness of $<'$ reduces in an elementary way to the well-foundedness of $<$.
- If $\alpha < f(\vec{\beta})$, then $\alpha < \beta_i$ for some i or $\alpha = f(\vec{\alpha})$ for some $\vec{\alpha} <' \vec{\beta}$.

We could show that the Bachmann-Howard ordinal is an upper bound for the order type of such systems and recently as well that this bound is sharp.

In this talk we will explore the result of replacing the set of arguments of f by more complicated structures and of using stronger principles in the reduction of $<'$ to $<$.

In a first step, this set of arguments will be essentially a set of terms OT_2 based on the ordinal notations OT to be defined and the reduction of $<'$ to $<$ follows by $(\mathrm{OT}_2, <')$ being an ordinal notation system from below in the original sense, but based on $(\mathrm{OT}, <)$. This construction will reach $\psi(\varepsilon_{\Omega_2+1})$ and can be regarded as "from below": the function f will still be inflating with respect to OT.

The next construction will be to add a third level $(\mathrm{OT}_3, <'')$ and by adding even more levels indexed by ordinals we will eventually reach the strength of KPI or $(\Delta^1_2 - \mathrm{CA}) + (\mathrm{BI})$.

These constructions are just another way of looking at ordinary ordinal notation systems which use collapsing functions and large cardinals or their recursive analogues. (We will only consider the levels up to one inaccessible cardinal, but it is only a matter of bureaucracy to extend this to systems using larger cardinals.) In this sense large cardinals are not needed for denoting these ordinals but are only used to define in a short way ordinal notation systems.

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We address proof-theoretic aspects of various kinds of transfinitely iterated fixed point