Corrections to my article "Ordinal Systems"

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Unfortunately Markus Michelbrink, Hannover, Prof. H. Schwichtenberg, Munich, and myself have detected some mistakes in the article

Anton Setzer, Ordinal systems, in Cooper, B. and Truss, J. (Eds.): *Sets and proofs*, Cambridge University Press, Cambridge, pp. 301 – 331, 1999

of which one is a crucial cut-and-paste error in the definition of ordinal systems.

A corrected version is available via my home page (see below). The list of corrections follows.

Abstract, line 6 – 7	Replace "transfinitely iterated fixed point theories $\lvert\widehat{\mathrm{ID}}_\sigma\rvert$" by "theories of transfinitely iterated inductive definitions $\lvert\mathrm{ID}_\sigma\rvert$".
Def. 1.1 (a)	For clarification insert the following text after the paragraph defining what a *class* is: "A *binary relation* $\prec$ is a class. For binary relations $\prec$ we define $r \prec s := \pi(r,s) \in \prec$, where π is a standard primitive recursive pairing function on the natural numbers having the usual properties." Add further before "*Transfinite induction over* $(A, \prec)$ *is in* PRA *reducible to transfinite induction over* $(A_i, \prec_i)$ $(i = 1, \dots, n)$, ..." the following sentence: "Let A be a class, $\prec$ be a binary relation, both depending on unary free predicates A_i and binary free predicates $\prec_i$ $(i = 1, \dots, n)$."
Def. 1.1 (b)	This part has to be rewritten as follows: "Assume B is a class and $\prec$ is a binary relation, both depending on unary free predicates A_i and binary free predicates $\prec_i$ $(i = 1, \dots, m)$. $(B, \prec)$ is an *elementary construction from* $(A_1, \prec_1)$, ..., $(A_m, \prec_m)$, if the following holds: the formulas defining B, $\prec$ are formulas of the language of PRA with bounded quantifiers only (ie. quantifiers of the form $\forall x < t$, $\exists x < t$); PRA^+ proves that, if $(A_i, \prec_i)$ are linear orderings $(i = 1, \dots, m)$, so is $(B, \prec)$; transfinite induction over $(B, \prec)$ is PRA-reducible to transfinite induction over $(A_i, \prec_i)$."
Def. 1.1 (j)	"$\bigcup\{X \subseteq \lvert A\rvert \mid (X, \prec) \text{ well-ordered }\}$" instead of "$\bigcap\{X \subseteq \lvert A\rvert \mid (X, \prec) \text{ well-ordered }\}$".
Def. 2.1 (d)	Replace in (OS 1) and (OS 3) "Arg" by "T".
Sect. 2.2, line 9/10	Replace "(more precisely the formula $\forall n \in \mathbb{N}(n \leq \underline{n} \wedge \forall x \in \mathrm{T}(x < \underline{n} \leftrightarrow \exists l < n.x = \underline{l}))$ is provable in PRA)." by "(more precisely the formula $\forall n \in \mathbb{N}(n \leq \underline{n} \wedge \forall x \in \mathrm{T}(x \prec \underline{n} \leftrightarrow \exists l < n.x = \underline{l}))$ is provable in PRA)."

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Lem. 2.2 (a), line 1/2	Omit "Let $b \in \mathrm{T}$".
Lem. 2.2 (a), line 4	Replace "$\forall\alpha, \beta < \gamma(\alpha < \beta \to a_\alpha \prec' a_\beta)$" by "$\forall\alpha, \beta < \gamma(\alpha < \beta \to a_\alpha \prec a_\beta)$".
Proof of Lem 2.2, (a), line 10	Replace "$\mathrm{k}(b) \prec b \prec a$" by "$\mathrm{k}(b) \prec b \prec a_\delta$".
Lem. 2.8 (a)	Replace "$\forall a \in \mathrm{T}'.\widehat{\mathrm{k}}(a) \subseteq \mathrm{T}$" by "$\forall a \in \mathrm{T}'.\widehat{\mathrm{l}}(a) \subseteq \widehat{\mathrm{k}}(a) \subseteq \mathrm{T}$".
Lem 2.8 (b) line 1	Replace "$\forall a \in \mathrm{T}'(f[\widehat{\mathrm{k}}(a)] = \mathrm{k}^0(f(a)) \wedge f[\widehat{\mathrm{l}}(a)] = \mathrm{l}'(f(a)))$" by "$\forall a \in \mathrm{T}'(f[\widehat{\mathrm{k}}(a)] = \mathrm{k}^0(f(a)) \wedge f[\widehat{\mathrm{l}}(a)] = \mathrm{l}(f(a)))$".